Pre-Calculus First Semester Review

Non-Calculator

For the following:
(a) Identify the parent
(b) Describe the transformation.
(c) Sketch the graph.

~~[1.5] 1. $f(x) = \dfrac{1}{x+2} - 3$~~

~~a) ________~~

~~b) ________~~

[1.4] 2. $f(x) = -2|x+3| + 1$

a) $y = |x|$

b) $(x-3,\ -2y+1)$

$0,0 \to (-3,1)$
$(-1,1) \to (-4,-1)$
$(1,1) \to (-2,-1)$

[1.5] 3. $f(x) = -2(x+1)^2 + 4$
[2.1]

a) $y = x^2$

b) $(x-1,\ -2y+4)$

Vertex: $(-1,4)$

Axis of symmetry: $x = -1$

$-2,4 \to (-3,-4)$
$-1,1 \quad (-2,2)$
$0,0 \to (-1,4)$
$1,1 \to (0,2)$
$2,4 \to (1,-4)$

[1.5] 4. $f(x) = x^2 + 8x + 11$
[2.1]

a) $y = x^2$

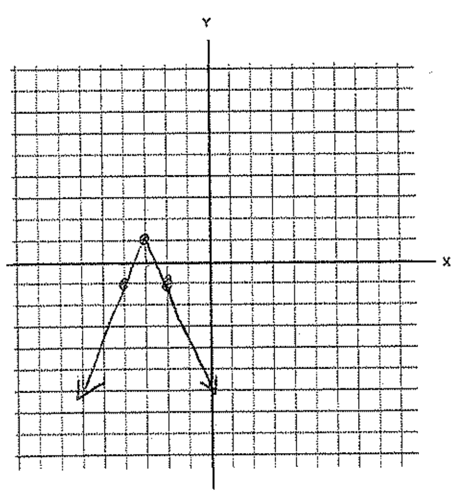
b) ________

Vertex: $x = \dfrac{-b}{2a} = -4 \qquad y = (-4)^2 + 8(-4) + 11 = -5$

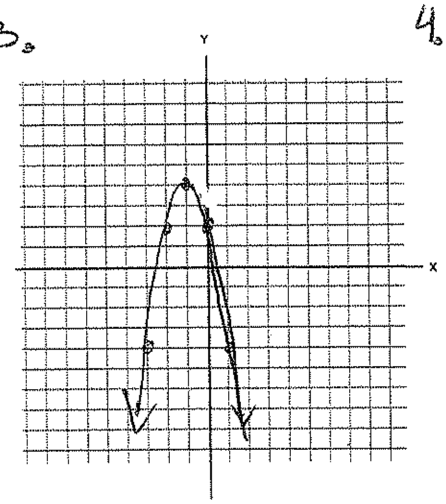
$(-4,-5)$

Axis of symmetry: $x = -4$

2.



3.



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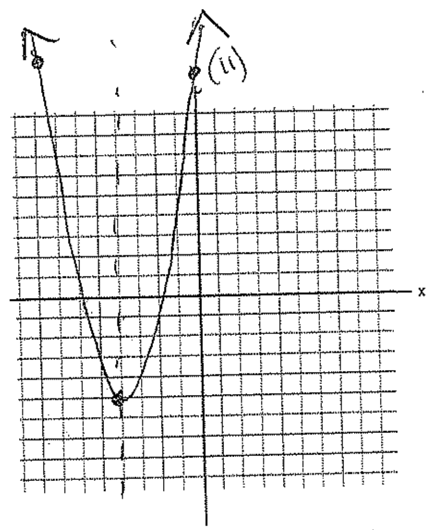


Solve. Check for extraneous roots.

[P3] 8. $2(5-2y)-3(1-y)\geq y+1$

$10-4y-3+3y\geq y+1$

$-2y\geq -6$

$y\leq 3$

$\boxed{(-\infty, 3]}$

[P3] 9. $2\left(\frac{x-2}{3}\right)+3\left(\frac{x+5}{2}\right)=2\left(\frac{1}{3}\right)$

$2x-4+3x+15=2$

$5x=-9$

$\boxed{x=-9/5}$

[P5] 10. $|2x-5|>4.2$

$2x-5>4.2 \qquad 2x-5<-4.2$

$2x>9.2 \qquad 2x<.8$

$x>4.6 \qquad x<.4$

$\boxed{(-\infty, .4)\cup(4.6, \infty)}$

[P5] 11. $|-x+4|-3\leq 7$

$|-x+4|\leq 10$

$-x+4\leq 10 \qquad -x+4\geq -10$

$-x\leq 6 \qquad -x\geq -14$

$x\geq -6 \qquad x\leq +14$

$\boxed{[-6, 14]}$

[P5] 12. $\frac{3x}{x+1}+\frac{5}{x-2}=\frac{15}{x^2-x-2}$

[2.7]

$x^2-x-2 = (x+1)(x-2)$

$(x-2)3x+(x+1)5=15$

$3x^2-6x+5x+5=15$

$3x^2-x-10=0$

$(3x+5)(x-2)=0$

$x=-5/3, \cancel{2}$

2 is an extraneous root

$\boxed{x=-5/3}$

[P5] 13. $4x^2-7x+5=0$

$x=\frac{7\pm\sqrt{49-4(4)(5)}}{2(4)}$

$=\frac{7\pm\sqrt{-31}}{8}$

$x=\frac{7\pm i\sqrt{31}}{8}$ no real roots

[P5] 14. $-3\leq 1-2x<7$

$-1 \quad -1 \quad -1$

$-4\leq -2x<6$

$2\geq x>-3$

$\boxed{(-3, 2]}$

[2.8] 15. $\frac{2x+1}{(x+3)(x-1)}\leq 0$

asymptote $=x=-3$ $x=1$

$2x+1=0$

$x=-1/2$

$\boxed{(-\infty,-3)\cup[-1/2, 1)}$

$\frac{2(-4)+1}{(-4+3)(-4-1)}=$ negative

$\frac{2(0)+1}{(0+3)(0-1)}=$ neg

$\frac{2(-1)+1}{(-1+3)(-1-1)}=$ positive

$\frac{2(2)+1}{(2+3)(2-1)}=+$

$- \quad + \quad - \quad +$

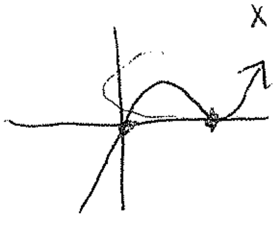
$-3 \quad -1/2 \quad 1$

[2.8] 16. $x^3-2x^2+x\geq 0$

$x(x^2-2x+1)\geq 0$

$x(x-1)(x-1)\geq 0$

$x=0, 1$ (mult 2)



section $\geq x=0$ is $\boxed{[0, \infty)}$

[2.8] 17. $\frac{2}{x+1}-\frac{3}{x-5}>0$

asymptote $\rightarrow x=-1, x=5$

$2(x-5)-3(x+1)=0$

$2x-10-3x-3=0$

$-13=x$

$+ \quad - \quad + \quad -$

$-13 \quad -1 \quad 5$

$\boxed{(-\infty,-13)\cup(-1,5)}$

[P1] Simplify. Express your answer without negative exponents.

18. $\frac{(uv^{-2})^{-3}}{u^{-5}v^{2}}$ $= \frac{u^{-3}v^{6}}{u^{-5}v^{2}} = \frac{u^{5}v^{6}}{u^{3}v^{2}} = \boxed{u^{2}v^{4}}$

19. $\frac{4a^3b}{a^2b^3} \cdot \frac{3b^2}{2a^2b^4}$ $= \boxed{\frac{6}{ab^4}}$

[P4] 20. Write a general form equation of a line a) parallel to and b) perpendicular to $5x - y = 7$ and passing through the point (3, –4).

$y = 5x - 7$
slope $= 5$

a) parallel slope $= 5$ $\quad y + 4 = 5(x - 3)$
$\boxed{y = 5x - 19}$

b) perp slope $= -1/5$ $\quad y + 4 = -1/5(x - 3)$
$y + 4 = -\frac{1}{5}x + \frac{3}{5}$
$5y + 20 = -x + 3$ $\rightarrow$ $x + 5y = -17$

[1.2] Find the domain. Express the answer in interval notation.

21. $f(x) = \sqrt{x^2 + 3}$ $\quad$ Domain $= \mathbb{R}$
$x^2 + 3 \geq 0$ $\quad (-\infty, \infty)$
$x^2 \geq -3$
all real #'s

22. $f(x) = \frac{\sqrt{x}}{x - 5}$ $\quad x \geq 0$
$x \neq 5$
$[0, 5) \cup (5, \infty)$

[1.3] Prove algebraically whether the function is even, odd, or neither.

23. $f(x) = 3x^3 - 2x$
$f(-x) = 3(-x)^3 - 2(-x)$
$= -3x^3 + 2x$
$f(-x) \neq f(x) \rightarrow$ not even
$-f(x) = -(3x^3 - 2x)$
$= -3x^3 + 2x$
$-f(x) = f(-x) \rightarrow \boxed{\text{ODD}}$

24. $f(x) = -2x^4 - 4x + 7$
$f(-x) = -2(-x)^4 - 4(-x) + 7$
$= -2x^4 + 4x + 7$
$f(-x) \neq f(x) \rightarrow$ not even
$-f(x) = -(-2x^4 - 4x + 7)$
$= +2x^4 + 4x - 7$
$-f(x) \neq f(-x) \rightarrow$ not odd
$\boxed{\text{NEITHER}}$

[1.4] Given $f(x) = (x - 4)^2$, $g(x) = 2x - 3$ and $h(x) = \sqrt{x + 5}$ Find and simplify the answer.

25. $f \circ h(4)$
$f(h(4))$ $\quad h(4) = \sqrt{4 + 5} = 3$
$f(3) = (3 - 4)^2 = (-1)^2 = \boxed{1}$

26. $g(f(x))$
$2(f(x)) - 3$
$= 2(x - 4)^2 - 3$
$= 2(x^2 - 8x + 16) - 3 = \boxed{2x^2 - 16x + 29}$

27. $f + g$
$(x - 4)^2 + 2x - 3$
$= x^2 - 8x + 16 + 2x - 3$
$= \boxed{x^2 - 6x + 13}$

28. fg
$(x - 4)^2(2x - 3)$
$= (x^2 - 8x + 16)(2x - 3)$
$= 2x^3 - 16x^2 + 32x - 3x^2 + 24x - 48$
$\boxed{2x^3 - 19x^2 + 56x - 48}$

[1.4] 29. Given: $f(x) = x^3 + 2$. Find $f^{-1}(x)$.

$y = x^3 + 2$

$x = y^3 + 2$

$x - 2 = y^3 \qquad y = \sqrt[3]{x-2} \longrightarrow f^{-1}(x) = \sqrt[3]{x-2}$

[1.4] 30. **Prove** that f and g are inverses of each other.

$f(x) = 2x + 8 \qquad g(x) = \dfrac{x-8}{2}$

$f(g(x)) = 2\left(\frac{x-8}{2}\right) + 8$
$= x - 8 + 8$
$= x$

$g(f(x)) = \dfrac{2x+8-8}{2}$
$= \dfrac{2x}{2}$
$= x$

$f(g(x)) = g(f(x)) = x$

[2.3] Describe the end behavior of the polynomial using **limit** notation.

31. $f(x) = -2x^3 + 4x^2 + 1$

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = \infty$

32. $f(x) = 3x^4 + x^2 - 5$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = \infty$

[2.3] Find the zeros of the function algebraically.

33. $f(x) = 3x^2 + 2x - 5$

$0 = (3x+5)(x-1)$

$3x + 5 = 0 \qquad x - 1 = 0$

$x = -5/3 \qquad x = 1$

34. $f(x) = x^3 - 36x$

$0 = x(x^2 - 36$

$0 = x(x-6)(x+6)$

$x = 0, \pm 6$

[2.4] Find the zeros of the function and write the function as a product of linear and irreducible quadratic factors all with real coefficients.

35. $f(x) = x^3 - x^2 - x - 2$, given that $x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -1 & -2 \\ & & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$x = \dfrac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} = \dfrac{-1 \pm \sqrt{-3}}{2} = \dfrac{-1 \pm i\sqrt{3}}{2}$

no more real roots

roots $\rightarrow 2, \dfrac{-1 \pm i\sqrt{3}}{2}$

$f(x) = (x-2)(x^2 + x + 1)$

36. $f(x) = x^4 + 3x^3 - 3x^2 + 3x - 4$, given that $x = 1$ and $x = -4$

$$\begin{array}{r|rrrrr} 1 & 1 & 3 & -3 & 3 & -4 \\ & & 1 & 4 & 1 & 4 \\ \hline -4 & 1 & 4 & 1 & 4 & 0 \\ & & -4 & 0 & -4 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

$x^2 + \cancel{x} = 0$

$x^2 = -1$

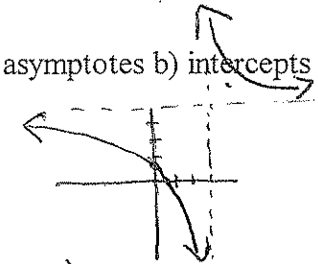
$x = \pm i$

roots $\rightarrow 1, -4, \pm i$

$f(x) = (x+4)(x-1)(x^2+1)$

[2.7] Find (if it exists) the a) asymptotes b) intercepts and c) domain of the function. Sketch the graph by hand.

37. $g(x) = \frac{4x-5}{x-3}$



a) asy → $x=3$ and $y=4$

b) x int → $4x-5=0$

$x = 4/5$

y int → $y = \frac{4(0)-5}{0-3} = \frac{5}{3}$

$g(4) = \frac{11}{1} = 11$

c) $(-\infty, 3) \cup (3, \infty)$

38. $g(x) = \frac{2x^2}{x^2-x-6}$ $(x-3)(x+2)$

a) asy → $x=3$ and $x=-2$

and $y = \frac{2x^2}{x^2} = 2$

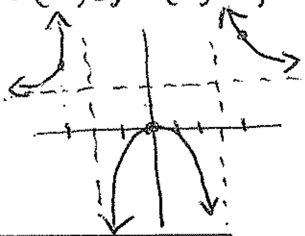
b) x int → $2x^2=0$ → $x=0$

y int → $y = \frac{2(0)^2}{0^2-0-6} = 0$

c) $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$

$g(-3) = \frac{18}{6} = 3$

$g(4) = \frac{16}{3} = 5\frac{1}{3}$



[2.5] 40. Write in $a + bi$ form: $\frac{2+4i}{3-2i}\left(\frac{3+2i}{3+2i}\right)$

$= \frac{6+4i+12i+8i^2}{9-4i^2} = \frac{-2+16i}{13} = \boxed{\frac{-2}{13}+\frac{16i}{13}}$

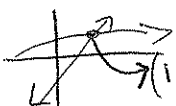
Graphing Calculator

[1.1] Solve by graphing.

41. $3x - 2 = \sqrt{x+4}$ $\boxed{x = 1.444}$

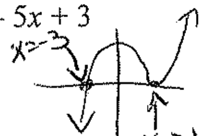
$Y_1 = 3x-2$

$Y_2 = \sqrt{x+4}$



(1.444, 2.333)

42. $0 = x^3 + x^2 - 5x + 3$ $\boxed{x = 1, -3}$

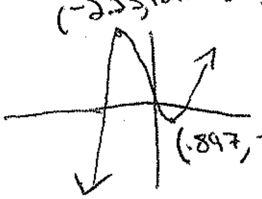


x=-3

x=1

[1.2] 43. Find all a) local maxima and minima and b) identify intervals on which the function is increasing, decreasing, or constant.

$f(x) = x^3 + 2x^2 - 6x$



(-2.23, 12.236)

(.897, -3.051)

a) 12.236 → local max

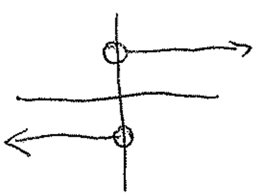
-3.051 → local min

b) increasing $(-\infty, -2.23) \cup (.897, \infty)$

decreasing $(-2.23, .897)$

[1.2] Graph the function and tell whether or not it has a point of discontinuity at $x = 0$. If there is a discontinuity, tell whether it is removable or non-removable.

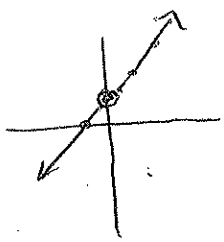
44. $f(x) = \frac{|x|}{x}$



yes, discontinuous @ x=0

non-removable

45. $h(x) = \frac{x^2+x}{x} = \frac{x(x+1)}{x} = x+1$

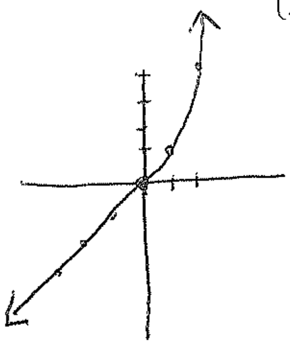


yes, discontinuous @ x=0

removable

[1.3] Sketch the graph of the piecewise-defined function. State whether the function is continuous or discontinuous at $x = 0$.

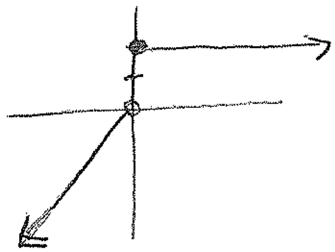
46. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$



continuous @ $x=0$

47. $f(x) = \begin{cases} -|x| & \text{if } x < 0 \\ 2 & \text{if } x \ge 0 \end{cases}$

discontinuous @ $x=0$



[2.1] 51. Write an equation for the linear function f with $f(-3) = -2$ and $f(4) = -8$. Express your answer in general form.

$(-3,-2)$ $(4,-8)$ $m = \frac{-6}{7}$

$y+2 = -\frac{6}{7}(x+3)$

$y+2 = -\frac{6}{7}x - \frac{18}{7}$

$7y + 14 = -6x - 18$

$\boxed{6x + 7y = -32}$

[2.1] 53. The table below gives the weight and pulse rate of selected mammals.

a) Write a power regression equation and state the power and constant of variation.

Mammal	Body Weight	Pulse Rate (beats/min)
Rat	0.2	420
Guinea Pig	0.3	300
Rabbit	2	205
Small Dog	5	120
Large Dog	30	85
Sheep	50	70
Human	70	72

STAT → CALC

PwrReg L_1, L_2, Y_1

$y = 231.2x^{-.297}$

b) Use the regression equation to determine the pulse rate of a human weighing 12 pounds.

$\boxed{Y_1(12) = 110.563 \text{ bts/min}}$

[2.4] Divide. Write a summary statement in polynomial form. Determine if the first polynomial is a factor of the second polynomial.

54. $2x+1;\ 6x^3-5x^2+9$

$$\begin{array}{r} 3x^2-4x+2+7/(2x+1) \\ 2x+1\,\overline{)\,6x^3-5x^2+0x+9} \\ -6x^3 \mp 3x^2 \\ \hline -8x^2+0x \\ +8x^2+4x \\ \hline 4x+9 \\ -4x+2 \\ \hline 7 \end{array}$$

no, not a factor

$$\frac{6x^3-5x^2+9}{2x+1} = 3x^2-4x+2+\frac{7}{2x+1}$$

55. $x-5;\ x^3-4x^2-7x+10$

$$\begin{array}{r|rrrr} 5 & 1 & -4 & -7 & 10 \\ & & 5 & 5 & -10 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$$\frac{x^3-4x^2-7x+10}{x-5} = x^2+x-2$$

yes it is a factor

[2.4 & 2.6] Find a polynomial equation with the given zeros. Express answers in standard form.

56. $\frac{1}{3}, -2, 5$

$(x+2)(x-5)(3x-1)$

$(x^2-3x-10)(3x-1)$

$\boxed{3x^3-10x^2-27x+10}$

57. a) $-1, 2-i$

$(x+1)(x-2-i)(x-2+i)$

$(x+1)(x^2-2x+\cancel{ix}-2x+4\cancel{-2i}\cancel{-ix}+\cancel{2i}-i^2)$

$(x+1)(x^2-4x+5)$

$\boxed{x^3-3x^2+x+5}$

b) $3, 4i$

$(x-3)(x+4i)(x-4i)$

$(x-3)(x^2-16i^2)$

$(x-3)(x^2+16)$

$\boxed{x^3-3x^2+16x-48}$